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FM-8066-ARPA  
JANUARY 1970

## SMALL-ANGLE SCATTERING OF LIGHT BY OCEAN WATER

H. T. Yura

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PREFACE

This Memorandum is part of Rand's study for the Advanced Research Projects Agency of those phenomena which affect the performance of underwater optical reconnaissance and guidance equipment. The objective of these studies is to provide sufficient understanding to permit the systems analyst to compute performance estimates under various operational conditions.

A better understanding of underwater visibility will be obtained only through an investigation of those mechanisms which give rise to the very intense small-angle forward scattering of light by ocean water. This Memorandum should be of interest to those concerned with the use of lasers in underwater detection and visibility.

SUMMARY

Small-angle ( $\lesssim 10^{-3}$  rad) scattering of light by ocean water is quantitatively analyzed. The two mechanisms which give rise to such scattering are suspended biological particles having an index of refraction close to that of water and refractive effects due to large-scale (compared to the laser beam diameter) index of refraction variations. Results of recent experiments performed at the Stanford Research Institute are compared with the present analysis and reasonable agreement is obtained. Also, the modulation transfer function (MTF) for the scattering mechanisms is given. It is found that for values of the transverse distances  $\rho$  less than the laser beam diameter the MTF due to the two mechanisms has a different functional dependence on transverse distance. Thus, an experimental determination of the dependence of the MTF on  $\rho$  will be useful in determining if a dominant scattering mechanism is at play. Also, we conclude that for long propagation paths ( $\gtrsim 10$  m) suspended particles can degrade the transverse coherence properties of the laser beam much more than do large-scale refractive index variations.

CONTENTS

PREFACE .....	iii
SUMMARY .....	v
Section	
I. INTRODUCTION .....	1
II. GENERAL REMARKS .....	4
III. SMALL-ANGLE SCATTERING DUE TO SUSPENDED PARTICLES .....	7
IV. REFRACTIVE EFFECTS DUE TO LARGE-SCALE INDEX VARIATIONS ..	10
V. TRANSVERSE COHERENCE PROPERTIES .....	13
VI. DISCUSSION .....	16
Conclusion .....	18
REFERENCES .....	19

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I. INTRODUCTION

The imaging properties of collimated laser beams traversing sea water have recently attracted considerable attention. Direct or indirect viewing is necessary to carry out underwater activities such as underwater target acquisition, guidance of submarines, and searching for metal ore nodules and marine specimens. The purpose of this Memorandum is to give a semi-quantitative analysis of those mechanisms which give rise to very intense small-angle scattering. A better understanding of underwater visibility will be obtained only through an investigation of those mechanisms which give rise to the very intense small-angle ( $\lesssim 10^{-3}$  rad) forward scattering of light by ocean water.

Duntley<sup>(1)</sup> has given an exhaustive account of light scattering by sea water for scattering angles greater than one degree. Recent experiments of small-angle scattering performed by researchers at the Stanford Research Institute indicate that the peak in the forward-scattering amplitude is orders of magnitude greater than that which would be obtained from an extrapolation of Duntley's data to zero degrees.\* The preliminary experiments performed by the SRI group indicate that two types of mechanism are responsible for the intense peak in forward-scattering amplitude: scattering from suspended particles such as transparent plankton with an index of refraction close to that of water, and scattering by refractive index variations in the water resulting from large-scale (some tens of centimeters) thermal and

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\* Private communication from G. Sorenson, Stanford Research Institute, Menlo Park, California.

saline variations. Strictly speaking, the second mechanism should be called a refractive effect rather than a scattering effect, since for all cases of interest the laser beam diameter  $d$  (defined as the beam diameter at the exit pupil of the transmitting optics (on the order of a few centimeters)) is much smaller than the characteristic size of the random refractive index variations (tens of centimeters) and hence lensing effects give rise to random tilts of the wavefronts.

The gross attenuation of a light beam by sea water is due to two mechanisms: scattering and absorption. The former refers to any process by which the direction of an individual photon is changed without any other alteration, while the latter refers to all of the many thermodynamically irreversible processes by which the energy that the photon carries is transformed into thermal kinetic energy, chemical potential energy, etc. Defining  $\alpha$  as the spectral volume attenuation coefficient, the radiant power  $P(L)$  reaching a distance  $L$  without having been deviated by any type of scattering and absorption process is given by

$$P(L) = P_0 e^{-\alpha L} \quad (1)$$

where  $P_0$  is the power in the transmitting aperture and it is understood that a monochromatic light wave is under consideration. The volume attenuation coefficient is the sum of the volume absorption coefficient  $a$  and the total volume scattering coefficient  $s$ . The attenuation coefficient  $\alpha$  is a function of frequency having a minimum\*

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\* The region near 0.5  $\mu$  in the electromagnetic spectrum is the only place where water is a relatively poor absorber. In the ultraviolet

value for  $\lambda = 0.48 \mu$  where  $\alpha^{-1} = 28 \text{ m}$  for distilled water.<sup>(1)</sup> For sea water,  $\alpha^{-1}$  is typically of the order of 10 m or less at this wavelength. The quantity  $\alpha$  thus determines the gross energetic properties of a collimated light beam as a function of propagation distance. The essential parameters that describe the small-angle scattering are the mean size, refractive index, and concentration of the suspended particle and the characteristic length and mean-square deviation of the refractive index variations of the sea water.

In Section II we make some general remarks concerning scattering in a random medium. In Section III we calculate the forward scattering off of suspended particles in the water which are assumed large compared to the wavelength, while in Section IV the deviations of the light beam are calculated due to large-scale refractive index variations. In Section V we calculate the modulation transfer function (MTF) associated with these two mechanisms. The MTF gives a complete statistical description of the transverse coherence properties of the beam as a function of propagation distance. Finally, in Section VI we give some numerical estimates of the MTFs that might be expected under various conditions.

It is assumed throughout that the time interval during which the measurements are performed is large enough so that the time average may be replaced by an ensemble average (denoted in the following by a bar over a quantity), and only horizontal propagation paths are considered.

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water absorbs strongly due to the excitations of its electronic levels, while in the infrared and far-infrared strong absorption takes place in the broad continuum of its vibrational and rotational states. In the microwave region of the spectrum, sea water will absorb strongly, since ionic conductivity is important there. It is extremely unlikely that any gaps other than that in the visible region exist (although the skin depth at long radio wavelengths can be large).

### II. GENERAL REMARKS

In the following we are dealing with scattering angles  $\theta \ll 1$ , where  $\theta$  is defined as the angle between the unperturbed wave number,  $k$ , which is assumed parallel to the  $x$ -axis at the exit pupil of the transmitting optics and the deviated wave number assumed to be of the same magnitude as  $k$  but differing in direction only. In traversing a region of sea water, a beam of light undergoes a number of multiple scatterings and partial absorptions. The effect of these absorptions and scatterings (for  $\theta \gtrsim$  a few times  $10^{-3}$  rad) at range  $L$  is taken into account in expressions involving the square of the field by the factor  $e^{-\alpha L}$ . This factor is to be understood throughout and is omitted in the following formulae. Also, we assume that the particle distribution and refractive index variations are randomly distributed and isotropic. From the assumption of isotropy it follows that the scattering amplitude is independent of the azimuthal angle  $\phi$ , and hence is only a function of  $\theta$  and  $L$ .

Since it is assumed that the scattering centers are randomly distributed and that the mean distance between them is large compared with their characteristic size, it is a good approximation also to assume that the photons after traversing a distance  $L$  have undergone a series of independent single-scattering events. From this assumption it follows from the Central Limit Theorem that for many scattering events that the probability of a wave vector having components  $k_y$  and  $k_z$  is given by

$$p = c \exp \left( -\frac{k_y^2}{2k_y^2} \right) \exp \left( -\frac{k_z^2}{2k_z^2} \right) \quad (2)$$

where  $c$  is a constant, and  $\overline{k_y^2}$  and  $\overline{k_z^2}$  are the mean-square components of the wave vector at propagation distance  $L$  along the  $y$ - and  $z$ -axis, respectively. The mean-square transverse wave numbers are related to their corresponding single-scattering quantities by (with a similar expression for  $\overline{k_z^2}$ )

$$\overline{k_y^2} = \sum_i N_i (\overline{k_y^2})_{si} \quad (3)$$

where  $N_i$  and  $(\overline{k_y^2})_{si}$  are the mean number of single scatterings and mean-square single-scattering wave number along the  $y$ -axis due to the  $i^{th}$  mechanism, respectively. The assumption of isotropy implies that  $\overline{k_y^2} = \overline{k_z^2}$ . The mean number of scatterings is given by

$$N_i = \frac{L}{L_i} \quad (4)$$

where  $L_i$  is the single-scattering mean free path for the  $i^{th}$  scattering mechanism. The mean-square scattering angle  $\overline{\theta^2}$  is given by

$$\overline{\theta^2} = \frac{\overline{k_y^2} + \overline{k_z^2}}{\overline{k^2}} \quad (5)$$

with a corresponding expression for the mean-square single-scattering angle.

In closing this section it is noted that the relationship between the normalized probability of single scattering into solid angle  $d\Omega$  and the differential single-scattering cross section is

$$p_1(\theta, \varphi) = \frac{d\sigma_1}{d\Omega} \frac{1}{\left( \frac{d\sigma_1}{d\Omega} \right) d\Omega} \quad (6)$$

and

$$L_i = \sigma_1^{-1}$$

### III. SMALL-ANGLE SCATTERING DUE TO SUSPENDED PARTICLES

Consider a random suspension of spherical particles of radius  $D$  which is assumed large compared with  $k^{-1}$ , where  $k$  is the optical wave number of the light in water ( $= 2\pi n_0 / \lambda$ , where  $n_0$  is the mean index of refraction of water at this wavelength). Furthermore, we assume that the particles are nonabsorbing with an index of refraction given by  $n_p = n_0 + \delta n$ , where  $|\delta n| \ll n_0$ , and that the relative volume concentration of particles is constant and denoted by  $f$ .

Considering the index of refraction of the medium  $n(\underline{r})$ , water plus suspended particles, we have

$$n(\underline{r}) = n_0 \left( 1 + \frac{f \delta n(\underline{r})}{n_0} \right) \quad (7)$$

with  $\delta n(\underline{r})$  as a random variable with statistics

$$\overline{\delta n(\underline{r})} = 0$$

$$\overline{\delta n(\underline{r}) \delta n(\underline{r} + \underline{R})} = \omega^2 B(R)$$

where the correlation function  $B(R)$  is assumed independent of angle,  $B(0) = 1$ , and  $\omega$  is the standard deviation of  $\delta n$ .

The single-scattering differential cross section from a medium with an index of refraction given by Eq. (7) is<sup>(2)</sup>

$$\frac{d\sigma_1}{d\Omega} = 2\pi k^4 \Phi_n \left( 2k \sin \frac{\theta}{2} \right) \quad (9)$$

where  $k = 2\pi n_0 / \lambda$  and

$$\phi_n(K) = \frac{f^2 \mu^2}{n_0^2} \int d^3 R B(R) e^{i K \cdot R} \quad (10)$$

In order to proceed we need an expression for the correlation function of the index fluctuations  $B(R)$ . Real sea water contains particles of various sizes and shapes and hence  $B(R)$  will, in general, be a complicated function of direction and have no single characteristic length associated with it. In order not to make the mathematics unduly difficult but still retain all of the essential physics of the scattering process, we choose a convenient analytic form for  $B(R)$  -- a spherically symmetric gaussian-shaped function with a single characteristic length. In order to include different size particles, we may let the relative volume concentration  $f$  be parametrically related to the size of the particle and average the results for a single size over a distribution of sizes.

With  $B(R) = \exp(-R^2/D^2)$ , we find from Eqs. (9) and (10)

$$\frac{d\sigma_1}{d\Omega} = \frac{\mu^2 f^2 D^3 k^4}{n_0^2 8\pi^{3/2}} \exp\left[-\frac{D^2 k^2 \theta^2}{4}\right] \quad (11)$$

where we have used the approximation  $\sin \theta \approx \theta$  in obtaining this result. From Eq. (11) we find that

$$\overline{\theta_1^2} = \frac{4}{(kD)^2} \quad (12)$$

The single-scattering mean free path for small-angle scattering is given by

$$\begin{aligned}
 L_1 &= c_1^{-1} \\
 &= \frac{2n_o^2}{\pi^{\frac{1}{2}} \mu_f^2 k^2 D} \quad (13)
 \end{aligned}$$

We note that the single-scattering mean path is inversely proportional to the square of the relative volume concentration.

From Eqs. (2) to (5) we find that the mean-square scattering angle of a light beam after propagating a distance  $L$  is given by

$$\begin{aligned}
 \overline{\theta^2} &= \frac{L}{L_1} \overline{\theta_1^2} \\
 &= \frac{2\sqrt{\pi} \mu_f^2}{n_o^2 D} L \quad (14)
 \end{aligned}$$

From Eq. (12) we find that  $\sqrt{\overline{\theta_1^2}} \approx 10^{-3}$  rad for  $D = 10^{-2}$  cm,  $\lambda = 0.5 \mu$ , and  $n_o \approx 1.33$ , while  $\sqrt{\overline{\theta_1^2}} \approx 10^{-4}$  rad for  $D = 10^{-1}$  cm and the same values of  $\lambda$  and  $n_o$ . A precise numerical value for  $L_1$  is more difficult to ascertain, since the numerical value of  $\mu_f^2$  is uncertain a priori for typical ocean water (although not difficult to measure experimentally for a given sample of ocean water). If we arbitrarily pick  $\mu_f^2 \sim 10^{-11}$ , we find that  $L_1 \sim 10$  m for  $D = 10^{-2}$  cm, while  $L_1 \sim 1$  m for  $D = 10^{-1}$  cm. With this value of  $\mu_f^2$  we find from Eq. (14) that  $\sqrt{\overline{\theta^2}} \sim 2 \times 10^{-3}$  rad for  $D = 10^{-2}$  and  $L = 50$  m, and  $\sqrt{\overline{\theta^2}} \sim 0.7 \times 10^{-3}$  rad for  $D = 10^{-1}$  cm and  $L = 50$  m.

As mentioned previously, an average of Eqs. (12) through (14) over a particle size distribution would give the effect of various sized organisms on the scattering. Due to lack of knowledge of the parameters  $\mu$  and  $f$  on particle size and geographic location, we do not feel that further calculations are warranted.

IV. REFRACTIVE EFFECTS DUE TO LARGE-SCALE INDEX VARIATIONS

On a larger scale (large compared to the beam diameter, which is typically of the order of a few centimeters), ocean water contains refractive inhomogeneities whose characteristic length "a" is of the order of tens of centimeters.<sup>(3)</sup> These inhomogeneities are caused by random temperature fluctuations having average temperature variations (about the mean) of the order 0.05 K. In addition, salinity variations in the ocean can produce index variations of the same order as do temperature variations. In the first approximation the effects of temperature and salinity are additive in their effect on the index of refraction. Not much is known about salinity variations (about the mean) for typical ocean water, and hence we focus our attention on temperature effects, pointing out below the modifications due to salinity variation.

For the purposes of describing the propagation of light through a medium characterized by such large-scale index variations as described above, we may envision the water volume to be composed of a number of subvolumes (cells) of characteristic length  $a$  and that the index of refraction is roughly constant over each cell (differing from  $n_0$  by an amount  $\overline{(\Delta n)^2}^{1/2}$ , where  $\overline{n^2}$  is the mean-square deviation of the index in the cell). The average number of cells traversed in a distance  $L$  is  $L/a$ .

A light beam of diameter  $d \ll a$  will be deflected as a whole by an angle  $\Delta\theta$  upon traversing through each cell. From Snell's law we find that  $\Delta\theta \approx \Delta n$ . Since it is assumed that  $\Delta n$  is a random variable with zero mean, we conclude that the mean-square angular deviation of the light beam as a whole upon traversing through  $L/a$  independent cells

is given by

$$\overline{(\Delta\theta)^2} = \frac{L}{a} \overline{(\Delta n)^2} \quad (15)$$

Assuming that the variations in the index of refraction are due to temperature variations, we have  $\Delta n \sim \Delta T/T$ , or, upon squaring and taking an ensemble average,

$$\overline{(\Delta n)^2} \sim \frac{\overline{(\Delta T)^2}}{T^2} \quad (16)$$

For  $\Delta T \sim 0.05^\circ K$ ,  $T = 300^\circ K$ ,  $a \sim 50$  cm, and  $L = 50$  m, we find from Eqs. (15) and (16) that  $\sqrt{\overline{(\Delta\theta)^2}} \sim 0.75 \times 10^{-3}$  rad. Recent experiments performed by researchers at SRI on small-angle scattering in San Francisco Bay and in Pacific coastal waters found a beam spread of the order  $10^{-4}$  rad for path lengths on the order of one meter at  $\lambda = 0.6328 \mu$ .\* The dominant scattering mechanism was thought to be either temperature or salinity variations. We note that a beam spread of the order of  $10^{-4}$  rad is obtained from Eqs. (15) and (16) for  $\Delta T \sim 0.02$  K,  $a = 50$  cm at a range of one meter. Thus the results of the SRI group could be explained on the basis of temperature variations alone. The effect of salinity variations is similar to Eq. (16) with  $\Delta T$  and  $T$  replaced by  $\Delta Cl$  and  $Cl$ , respectively, and the corresponding mean-square angular deviation is obtained by adding this term to the right-hand side of Eq. (15). It is concluded that for the experiments performed by the SRI group, salinity effects were

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\* Private communication from G. Sorenson, Stanford Research Institute, Menlo Park, California.

at most equal to the temperature effects. That is not to say that salinity effects are, in general, the lesser of the two. We must wait for further experimental checks on this point.

In an arbitrary volume of ocean water it is difficult to predict a priori whether scattering from suspended particles or refractive effects due to large-scale index variations is the dominant mechanism. Indeed, they may very well lead to a beam spreading of the same order of magnitude, which would make the analysis of the scattering phenomena more difficult.

In the next section we derive expressions for the MTF of the beam due to the two scattering mechanisms just discussed. It will be shown that for values of the transverse distance  $\rho$  of interest (on the order of a few millimeters to a centimeter) these two mechanisms yield MTFs which have different dependencies on  $\rho$ . Thus, an experimental determination of the MTF as a function of  $\rho$  will be useful in determining if a dominant scattering mechanism is at play.

### V. TRANSVERSE COHERENCE PROPERTIES

The modulation transfer function is defined here as the auto-correlation function of the complex field in the transverse direction at range L. This is the central quantity in determining the limiting resolution obtainable in forming an image through an inhomogeneous medium. We have

$$M(\rho) \equiv \overline{U(\underline{r}_1) U^*(\underline{r}_2)} \quad (17)$$

where  $\underline{r}_1$  and  $\underline{r}_2$  are two vectors in a plane transverse to the x-axis at distance L from the source, and  $\rho = |\underline{r}_1 - \underline{r}_2|$ . In all cases of interest the MTF is a function of  $\rho = |\underline{r}_1 - \underline{r}_2|$ .

The MTF due to scattering by suspended particles having a gaussian-shaped correlation function is<sup>(2)</sup>

$$M(\rho) = |U_0|^2 \exp \left[ - \frac{2L}{L_1} \left( 1 - e^{-\rho^2/D^2} \right) \right] \quad (18a)$$

$$\rightarrow |U_0|^2 \exp \left[ - \frac{2L\rho^2}{L_1 D^2} \right] \quad \text{for } \rho \ll D \quad (18b)$$

$$\rightarrow |U_0|^2 \exp \left[ - \frac{2L}{L_1} \right] \quad \text{for } \rho \gg D \quad (18c)$$

where  $U_0$  is a constant, D is the characteristic radius of the suspended particles (considered here to be  $\lesssim 1$  mm), and  $L_1$  is given by Eq. (13).

The MTF due to the deviations of the beam by large-scale (compared to beam diameter) refractive variations is obtained by noting that the

effect of these inhomogeneities is to cause a tilt of the wavefront by an amount  $\Delta\theta$  in. Specifically, the wavefront at the receiver plane is described by the function

$$\begin{aligned} u(\underline{r}) &= U_0 e^{i \underline{k} \cdot \underline{r}} \\ &= U_0 \exp \left\{ i(k_x x + k_y y + k_z z) \right\} \end{aligned} \quad (19)$$

where  $x$  and  $L$  are used interchangeably,  $U_0$  is a constant, and  $k = (k_x^2 + k_y^2 + k_z^2)^{\frac{1}{2}}$  is the magnitude of the optical wave vector. Due to the random nature of the inhomogeneities, the components  $k_y$  and  $k_z$  are independent random variables with zero mean (due to symmetry).\* Furthermore, interest is in small-angle scattering only where  $|k_y|, |k_z| \ll |k_x| \approx k$ .

From Eqs. (17) and (19) we obtain

$$\begin{aligned} M(\rho) &= |U_0|^2 \overline{\exp \left\{ i[k_y(y_1 - y_2) + k_z(z_1 - z_2)] \right\}} \\ &= |U_0|^2 \left[ \overline{e^{ik_y(y_1 - y_2)}} \right] \left[ \overline{e^{ik_z(z_1 - z_2)}} \right] \end{aligned} \quad (20)$$

since  $k_y$  and  $k_z$  are independent random variables. To compute the averages indicated in Eq. (20) we note that if  $y = e^f$ , where  $f$  is a normally distributed random variable with zero mean, then  $\bar{y} = \overline{e^f} = \exp \left\{ \frac{1}{2} \overline{f^2} \right\}$ . Applying this relation to Eq. (20), we obtain

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\* The random variable  $k_x$  is related to  $k_y$  and  $k_z$  through the relation  $k_x = (k^2 - k_y^2 - k_z^2)^{\frac{1}{2}}$ . Furthermore, the random variables  $k_y$  and  $k_z$  will, to a good approximation, be normally distributed, since they result from many independent single-scattering events.

$$M(\rho) = |U_0|^2 \left( e^{-\frac{1}{2} (y_1 - y_2)^2 \frac{k_y^2}{k_y^2}} \right) \left( e^{-\frac{1}{2} (z_1 - z_2)^2 \frac{k_z^2}{k_z^2}} \right)$$

Owing to the assumption of isotropy,  $\frac{k_y^2}{k_y^2} = \frac{k_z^2}{k_z^2} = \frac{k^2 \theta^2}{2}$  (where we have used Eq. (5)) and hence

$$M(\rho) = |U_0|^2 e^{-\frac{\rho^2 k^2 \theta^2}{4}} \quad (21)$$

where  $\rho^2 = (y_1 - y_2)^2 + (z_1 - z_2)^2$ .

Substituting Eq. (15) into Eq. (21) we obtain

$$M(\rho) = |U_0|^2 e^{-\frac{\rho^2 k^2 \Delta n^2 L}{4a}} \quad (22)$$

where  $\overline{\Delta n^2}$  is the rms index of refraction variation,  $a$  is the characteristic scale length of the inhomogeneities (assumed much greater than the beam diameter), and  $L$  is the propagation distance.

We note that if the  $\rho$  values of interest satisfy the condition  $D \ll \rho_{\text{interest}} \ll a$ , Eq. (18c) is applicable for the MTF due to scattering from suspended particles. By comparing Eqs. (18c) and (22), we conclude that the two scattering mechanisms discussed here yield an MTF with different functional dependencies on  $\rho$  in the range of interest. Thus, an experimental determination of the dependence of the MTF on  $\rho$  will be useful in determining if a dominant scattering mechanism is at play.

## VI. DISCUSSION

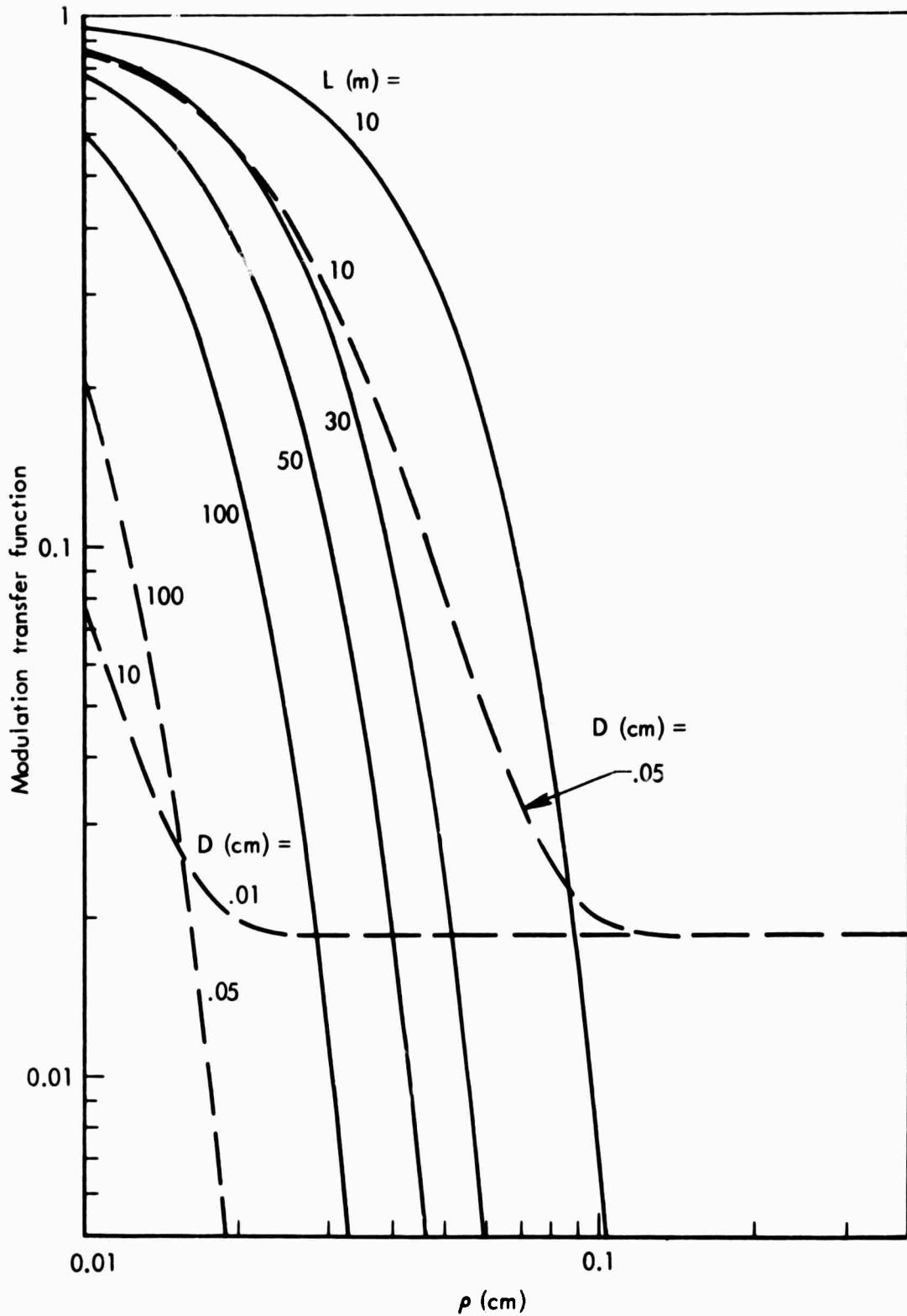
In this section we give some numerical estimates of the MTFs due to suspended particles and large-scale refractive index variations that might be expected under various conditions.

Any detecting system will have an effective noise level such that if the MTF is less than some number  $m_0$ , resolution is not possible. For example, the minimum useful contrast of photographic film lies between  $10^{-3}$  and  $10^{-2}$ . Defining  $\rho_{\max}$  such that for a fixed propagation distance  $L$ ,  $M(\rho \geq \rho_{\max}) \leq m_0$ , we note that the minimum distance resolvable in the object plane  $\Delta x_{\min}$  is related to the maximum transverse distance in the receiving aperture  $\rho_{\max}$

$$\Delta x_{\min} = \frac{L}{k\rho_{\max}} \quad (23)$$

The figure gives the MTF due to large-scale index variations as a function of transverse distance for various propagation paths. For the values of  $\rho$  shown, the MTF due to suspended particles for various values of  $D$  ( $L_1 = 5$  m) are indicated by the dashed lines. We note that if the MTF is independent of  $\rho$ , then  $\rho_{\max}$  is given by the laser beam diameter.

As a numerical example, consider the case where  $k = 10^5 \text{ cm}^{-1}$ ,  $\overline{\Delta n}^2 \sim 10^{-8}$ , large-scale refractive index variations are approximately 50 cm, and the single-scattering length due to suspended biological particles  $L_1$  is on the order of a few meters. Taking  $m_0 \sim 10^{-2}$ , we find from Eq. (18c) that the MTF due to suspended particles is  $\gtrsim 10^{-2}$  for  $L \sim 9$  m (we have taken  $L_1 \sim 5$  m). Hence, for propagation



Modulation transfer function due to large-scale ( $a = 50$  cm) index of refraction fluctuations (solid curve) and due to suspended particles ( $L_1 = 5$  m) (dashed curve)

distances greater than 10 m suspended biological particles seriously degrade resolution. On the other hand, from Eq. (22) we find that for  $L = 100$  m,  $\rho_{\max} = 1/30$  cm, and hence, from Eq. (23) we find that the minimum resolvable distance at 100 m is 3 cm. It is seen that for long-distance propagation ( $L \gtrsim 10$  m), the presence of suspended particles presents a much more serious limitation to resolution than do large-scale refractive index variations.

#### CONCLUSION

We have shown that the two different scattering mechanisms under certain analytical restrictions lead to different functional forms of the MTF. The need at present seems to be experimental measurements of the MTF to see if an experimental discrimination between the scattering mechanisms is possible on this basis.

REFERENCES

1. Duntley, S. J., "Light in the Sea," J. Opt. Soc. Amer., Vol. 53, No. 2, February 1963, pp. 214-233.
2. Tatarskii, V. I., Wave Propagation in a Turbulent Medium, McGraw-Hill Book Company, Inc., New York, 1961.
3. Lieberman, L., "The Effect of Temperature Inhomogeneities in the Ocean on the Propagation of Sound," J. Acoust. Soc. Amer., Vol. 23, No. 5, September 1951, pp. 563-570.

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10. ABSTRACT  <p>A semi-quantitative analysis of the mechanisms involved in very intense, small-angle forward scattering of light by ocean water. Two important mechanisms described are (1) suspended biological particles in the water having an index of refraction close to that of water and (2) deviations of the light beam due to large-scale refractive index variations. When the modulation transfer function (MTF) associated with these mechanisms is calculated, a complete statistical description is obtained for the transverse coherence properties of the beam as a function of the propagation distance. Some numerical estimates are given of the MTFs that might be expected under various conditions. Results of recent experiments performed at the Stanford Research Institute are compared with the present analysis and reasonable agreement is obtained.</p>		11. KEY WORDS  Wave propagation Oceanography Light Reconnaissance Detection Lasers